CS 112: Modeling Uncertainty in Information Systems

Prof. Jenn Wortman Vaughan

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Lecture 4

Reminders & Announcements

- Homework 1 is due in class this coming Monday
- Check Piazza for an easier solution to the third part of Monday's exercise
- Looks unlikely that the class size will be increased – talk to the OASA counselors if you're worried about meeting graduation requirements

Today

- The counting principle
- Permutations
- Combinations
- Binomial probabilities

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How can we count the items in these sets?

The Counting Principle

Consider a process consisting of r stages. Suppose that

- There are n_1 possible results at the first stage.
- Regardless of what happens at the first stage, there are n_2 possible results at the second stage.
- More generally, regardless of what happens at stage i-1, there are n_i possible results at stage i.

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Then the total number of possible results for the whole process is

$$n_1 \, n_2 \, n_3 \, \ldots \, n_r$$

The Counting Principle

- Suppose we have a set S of n elements. How many subsets of S are there?

The Counting Principle: Permutations

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- How many ways can we assign n threads to k processors in such a way that each processor is assigned exactly one thread and no thread is assigned to multiple processors? (Assume that $n \geq k$.)

The Counting Principle: Permutations

- Suppose that you have four books about math, ten books about computer science, and two books about physics. How many ways can you arrange these books on a shelf such that all of the books of a given subject are together?

The Counting Principle: Combinations

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- Suppose there are ten computer science courses being offered. How many ways can you create a schedule of four courses?

The Counting Principle: Combinations

When order doesn't matter, the number of ways to choose k elements from a set of n is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

pronounced "n choose k"

Disk Failure

A system contains $2x$ disks divided into x pairs where each pair of disks contain the same data. If one of the disks in a pair fails, the data can still be recovered, but if both disks fail, it cannot.

Suppose two “random” disks fail.

What is the probability that some data is inaccessible?

Poker Hands

Consider a standard deck of 52 cards in which each card has one of four suits (hearts, diamonds, spades, or clubs) and one of 13 ranks (ace, 2, 3, …, 10, jack, queen, or king)

- Suppose you draw five cards uniformly at random from a standard deck of cards. What is the probability of drawing four cards of the same rank (four kings, four threes, etc.)?

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- What is the probability of drawing five cards of the same suit (five hearts, five diamonds, etc.)?

Sequences of Independent Trials

- Suppose we have a biased coin that lands on heads with probability p. If we flip this coin five times, what is the probability that we see heads, heads, tails, tails, heads?

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- Suppose we have a biased coin that lands on heads with probability p. If we flip this coin five times, what is the probability that we see heads, heads, tails, tails, heads?

- If we flip this coin five times, what is the probability that we see exactly three heads and two tails?

Binomial Probabilities

Consider a sequence of n independent trials, each with a "success" probability of p. The probability of any particular sequence with k successes is

$$p^k(1-p)^{n-k}$$

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The probability that exactly k trials succeed is

$$\binom{n}{k}p^k(1-p)^{n-k}$$

Sequences of Independent Trials

A cell phone provider has enough resources to handle up to r data requests at once. Assume that every minute, each of the provider's n customers makes a data request with probability p, independent of what the other customers do.

What is the probability that exactly x customers will make a data request during a particular minute?

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What is the probability that more than r customers will make a data request during a particular minute?