CS 112: Modeling Uncertainty in Information Systems

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April 9, 2012

Lecture 3

Reminders & Announcements

Homework 1 is due in class on Monday, April 16.

You should be able to do the first five problems after today's lecture.

Today

- Independence of events
- Pairwise independence vs. independence of multiple events
- Conditional independence

Independence

Let's go back to the two-coin example again…

What is the probability that the first coin lands on tails given that the second coin lands on tails?

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What is the probability that the first coin lands on tails given that the second coin lands on tails?

Knowing whether or not the first coin lands on tails gives us no new information about how likely it is that the second coin lands on tails – these events are independent

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$$P(A \cap B) = P(A)\,P(B)$$

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To prove or disprove independence, it is enough to prove or disprove any one of these conditions!

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Independence

- Consider an event A. Are A and A^c independent?
- Suppose that A and B are independent. Are A and B^c independent?
- Suppose we flip two fair coins. Is the event that the first flip is tails independent from the event that the two flips are different?

Independence of Multiple Events

- Events A, B, and C are independent if and only if

$$P(A \cap B) = P(A)\,P(B)$$
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UPDATE: This definition can be generalized to any number of events… See the bottom of page 38.

Independence of Multiple Events

Back to the two-coin example…

$T_1 = \{TH, TT\}$ "1st toss is a tail"

$T_2 = \{HT, TT\}$ "2nd toss is a tail"

$D = \{HT, TH\}$ "the tosses have different results"

Are T_1, T_2, and D pairwise independent?

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Conditional Independence

- Events A and B are conditionally independent given C if and only if

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Conditional Independence

Back to the two-coin example again…

$T_1 = \{TH, TT\}$ "1st toss is a tail"

$T_2 = \{HT, TT\}$ "2nd toss is a tail"

$D = \{HT, TH\}$ "the tosses have different results"

We already know that T_1 and T_2 are independent.

Are they conditionally independent given D?

Exercise: Noisy Communication

A source transmits a message through a noisy channel. Each symbol is 0 with probability p or 1 with probability 1-p. Each 0 or 1 is received incorrectly (flipped) with probability ε_0 and ε_1 respectively, independent of other errors.

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For each part of the problem, first write down all of the information you are given. You will need to define some events to do so. Make sure you define them clearly!

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1) A single symbol is transmitted. What is the probability it is transmitted correctly?
2) If the sequence 1011 is transmitted, what is the probability that it is received correctly? (Hint: You can think of 1011 as fixed, not part of the experiment.)
3) The symbol 1 is received. What's the probability that it is correct? (It might help to write the sample space.)