CS 112: Modeling Uncertainty in Information Systems

Prof. Jenn Wortman Vaughan
June 6, 2012
Lecture 18
Reminders & Announcements

• Homework 5 is due in section this Friday, June 8

• Course evaluations are available for you to complete online – we’ll end a little early today in case you want to use that time to complete them
Resources for the Final Exam

• The final exam from 2011 has been posted on Piazza. The length, scope, and style of this year’s will be similar. Solutions will be posted after Friday’s sections.

• Friday’s sections will be review for the final, so bring any questions that you have.

• Prof. Vaughan will have her usual office hours tomorrow (Thursday), 11am-noon.

• On Monday, Jacob will hold office hours 11am-1pm and Prof. Vaughan will hold office hours 3-4pm.
Back to Absorption…
Absorption Probability Equations

**Theorem:** Consider a Markov chain in which each state is either absorbing or transient. Fix a particular absorbing state $s$. Then the probabilities $a_i$ of eventually reaching state $s$ after starting at state $i$ are the unique solutions to the following system of equations:

\[
\begin{align*}
    a_s &= 1 \\
    a_i &= 0 \quad \text{for all absorbing } i \neq s \\
    a_i &= \sum_{j=1}^{m} p_{i,j} a_j \quad \text{for all transient } i
\end{align*}
\]
Absorption Probability Equations

• What if we would like to calculate the probability of entering a multi-state recurrent class?
Absorption Probability Equations

• What if we would like to calculate the probability of entering a multi-state recurrent class?

• By merging states, we are able to apply the same ideas…
Time to Absorption
Time to Absorption

Every day in LA is either sunny or cloudy. If it is sunny one day, it will be sunny the following day with probability 0.75. If it is cloudy one day, it will be cloudy the next day with probability 0.5.
Time to Absorption

Every day in LA is either sunny or cloudy. If it is sunny one day, it will be sunny the following day with probability 0.75. If it is cloudy one day, it will be cloudy the next day with probability 0.5.

Assuming it is sunny today, how long must we wait on expectation until we’ve seen two consecutive cloudy days?
Expected Time to Absorption

- Let $\mu_i$ be the expected number of transitions until absorption starting at state $i$, i.e.,

$$\mu_i = \mathbb{E}[\min\{n \mid X_n \text{ is recurrent}\} \mid X_0 = i]$$

- What do we know about these values?
**Expected Time to Absorption**

**Theorem:** Let $\mu_i$ be the expected number of transitions until absorption starting at state $i$. The values $\mu_i$ for each state $i$ are the unique solutions to the following system of equations:

$$
\mu_i = 0 \quad \text{for all recurrent } i
$$

$$
\mu_i = 1 + \sum_{j=1}^{m} p_{i,j} \mu_j \quad \text{for all transient } i
$$
Time to Absorption

Every day in LA is either sunny or cloudy. If it is sunny one day, it will be sunny the following day with probability 0.75. If it is cloudy one day, it will be cloudy the next day with probability 0.5.

Assuming it is sunny today, how long must we wait on expectation until we’ve seen two consecutive cloudy days?
Snakes and Ladders

- Start at square 1
- On each turn, flip a fair coin; move 1 square if heads, 2 if tails
- If you land at the foot of a ladder, move up
- If you land at the head of a snake, move down
- What is the expected number of turns to get to square 9?
Final Reminders…

• Fill out your course evaluation online by Friday

• Bring questions to section on Friday

• Come to the extra office hours Monday if you have more questions (Jacob: 11am-1pm, Prof. Vaughan: 3-4pm)

Good luck on the final, and enjoy the summer!