CS 112: Modeling Uncertainty in Information Systems

Prof. Jenn Wortman Vaughan

June 6, 2012

Lecture 18

Reminders & Announcements

- Homework 5 is due in section this Friday, June 8
- Course evaluations are available for you to complete online – we'll end a little early today in case you want to use that time to complete them

Resources for the Final Exam

- The final exam from 2011 has been posted on Piazza. The length, scope, and style of this year's will be similar. Solutions will be posted after Friday's sections.
- Friday's sections will be review for the final, so bring any questions that you have.
- Prof. Vaughan will have her usual office hours tomorrow (Thursday), 11am-noon.
- On Monday, Jacob will hold office hours 11am-1pm and Prof. Vaughan will hold office hours 3-4pm.

Back to Absorption…

Absorption Probability Equations

Theorem: Consider a Markov chain in which each state is either absorbing or transient. Fix a particular absorbing state s. Then the probabilities a_i of eventually reaching state s after starting at state i are the unique solutions to the following system of equations:

$$a_s = 1$$

$$a_i = 0 \qquad \text{for all absorbing } i \neq s$$

$$a_i = \sum_{j=1}^{m} p_{i,j} a_j \qquad \text{for all transient } i$$

Absorption Probability Equations

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- By merging states, we are able to apply the same ideas…

Time to Absorption

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Assuming it is sunny today, how long must we wait on expectation until we've seen two consecutive cloudy days?

Expected Time to Absorption

- Let μ_i be the expected number of transitions until absorption starting at state i, i.e.,

$$\mu_i = \mathrm{E}[\min\{n \mid \mathrm{X}_n \text{ is recurrent}\} \mid \mathrm{X}_0 = i]$$

- What do we know about these values?

Expected Time to Absorption

Theorem: Let μ_i be the expected number of transitions until absorption starting at state i. The values μ_i for each state i are the unique solutions to the following system of equations:

$$\mu_i = 0 \qquad \text{for all recurrent } i$$

$$\mu_i = 1 + \sum_{j=1}^{m} p_{i,j}\mu_j \qquad \text{for all transient } i$$

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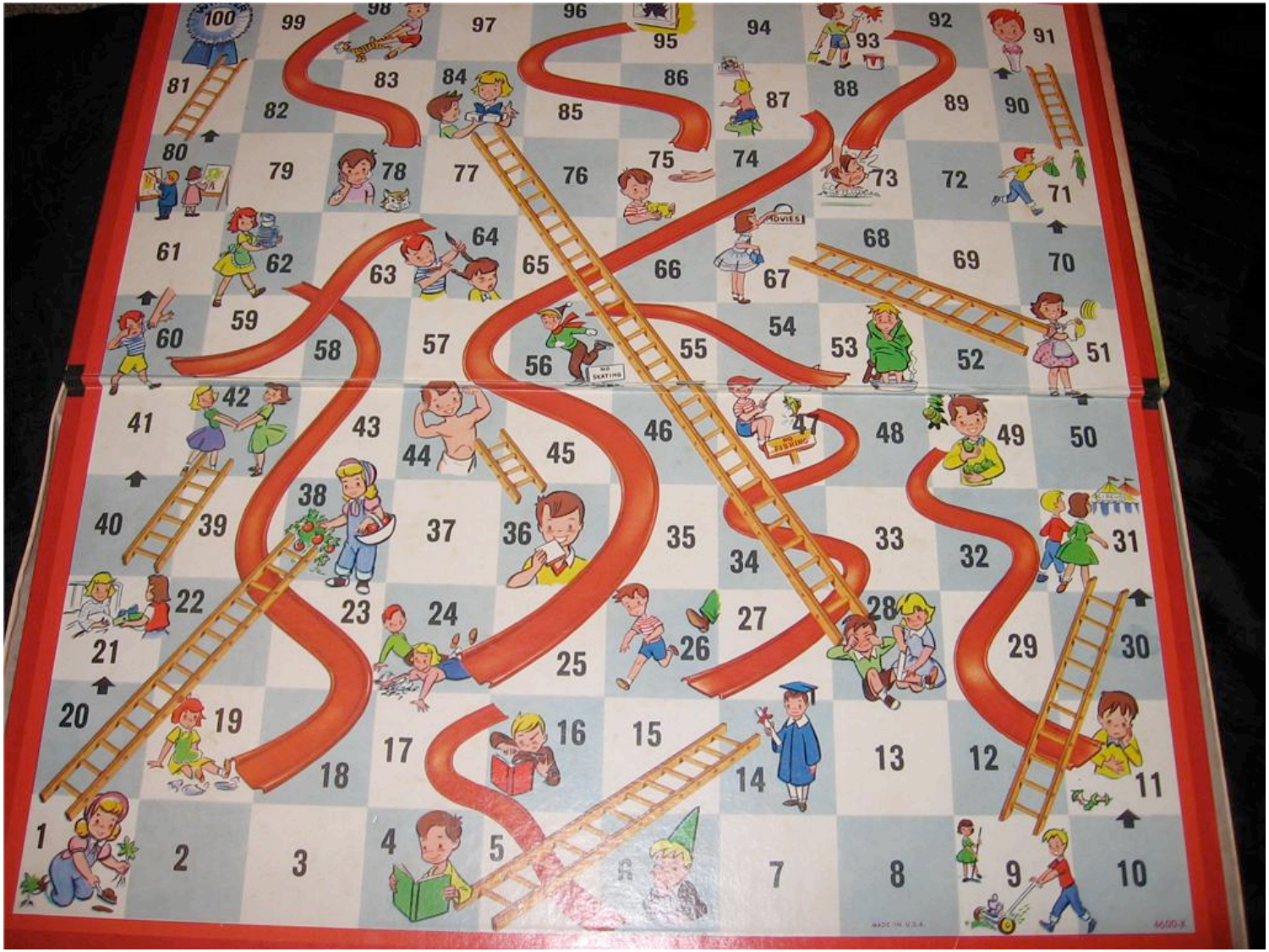
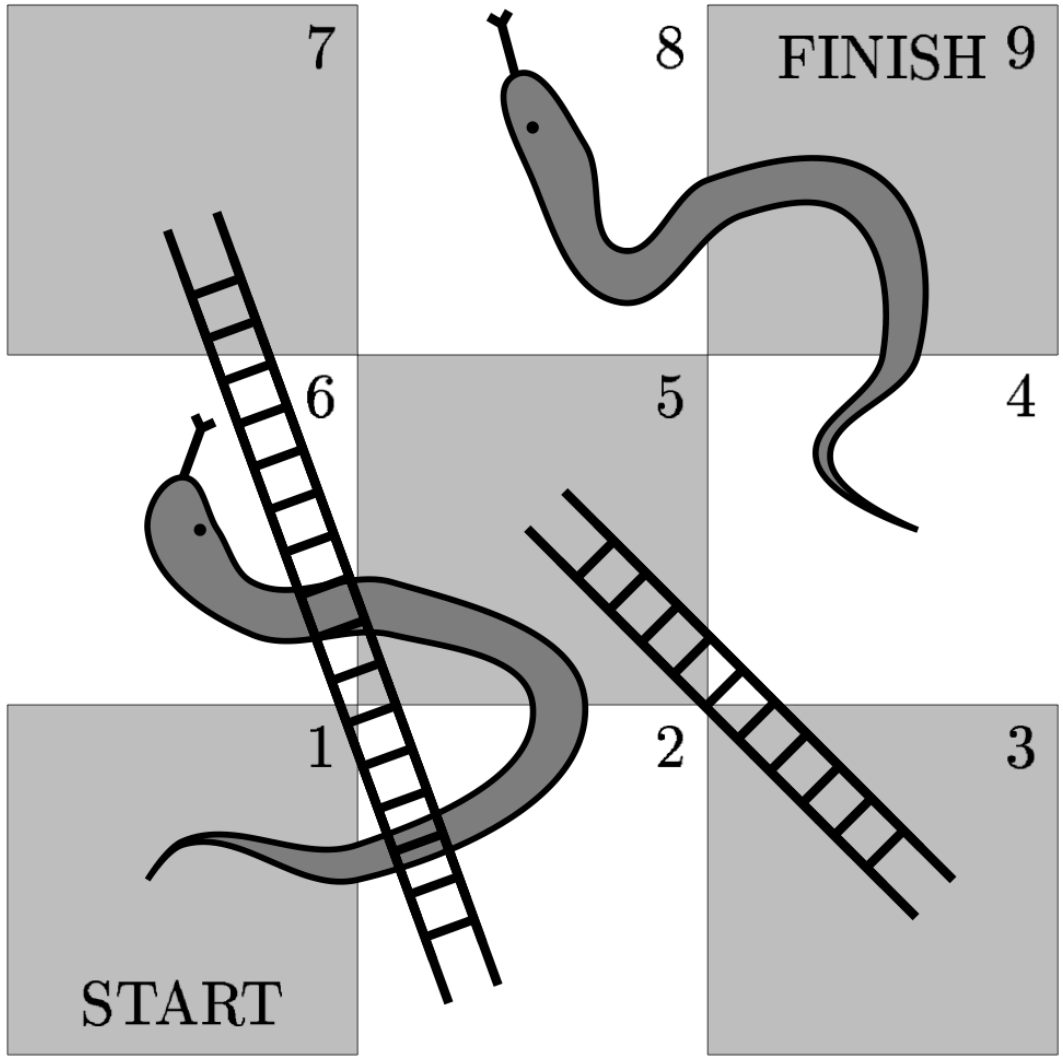
Snakes and Ladders



Snakes and Ladders

- Start at square 1
- On each turn, flip a fair coin; move 1 square if heads, 2 if tails
- If you land at the foot of a ladder, move up
- If you land at the head of a snake, move down
- What is the expected number of turns to get to square 9?



Final Reminders…

- Fill out your course evaluation online by Friday
- Bring questions to section on Friday
- Come to the extra office hours Monday if you have more questions (Jacob: 11am-1pm, Prof. Vaughan: 3-4pm)

Good luck on the final, and enjoy the summer!