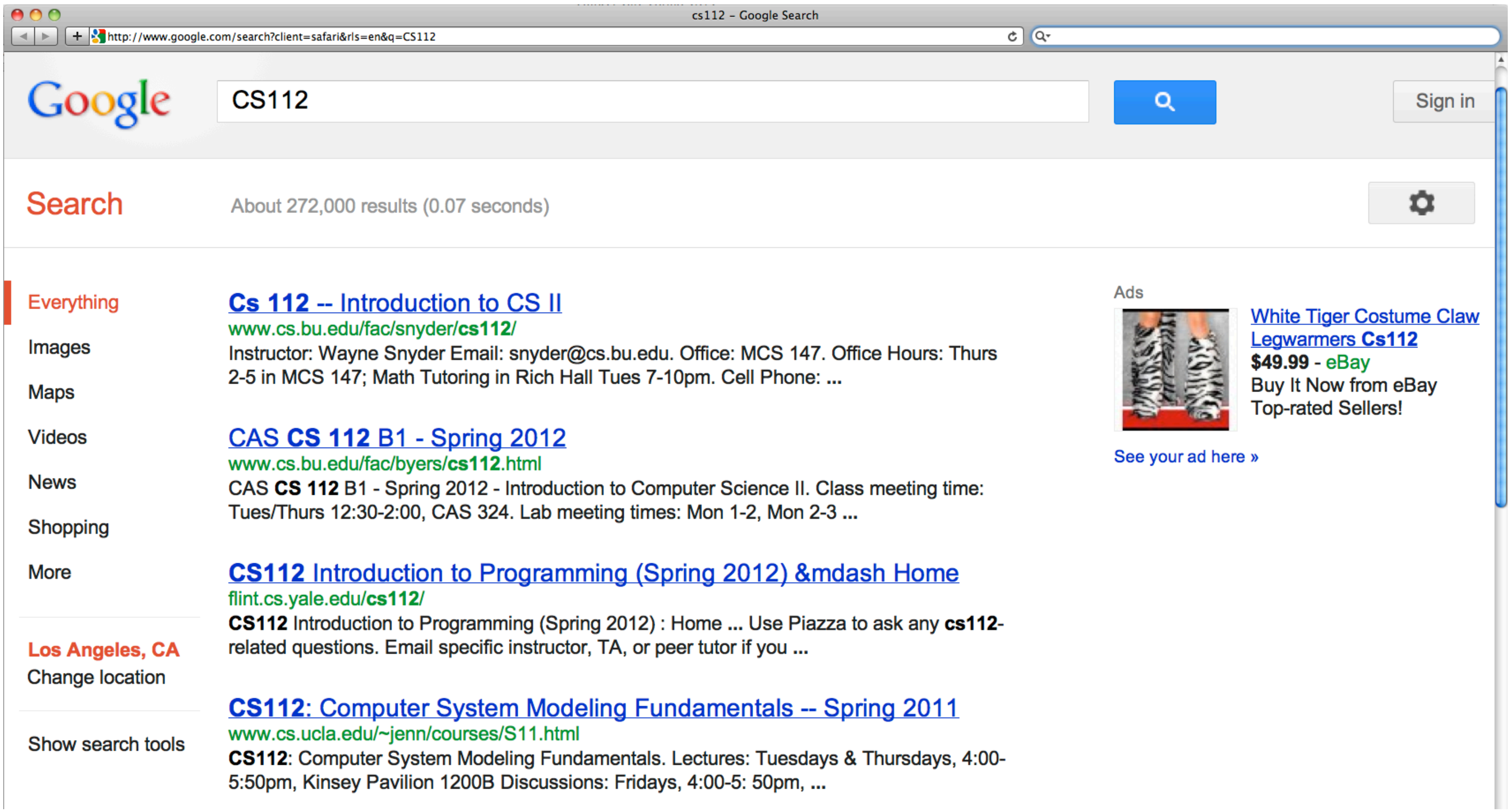
CS 112: Modeling Uncertainty in Information Systems

Prof. Jenn Wortman Vaughan

April 2, 2012

Uncertainty in Computer Science



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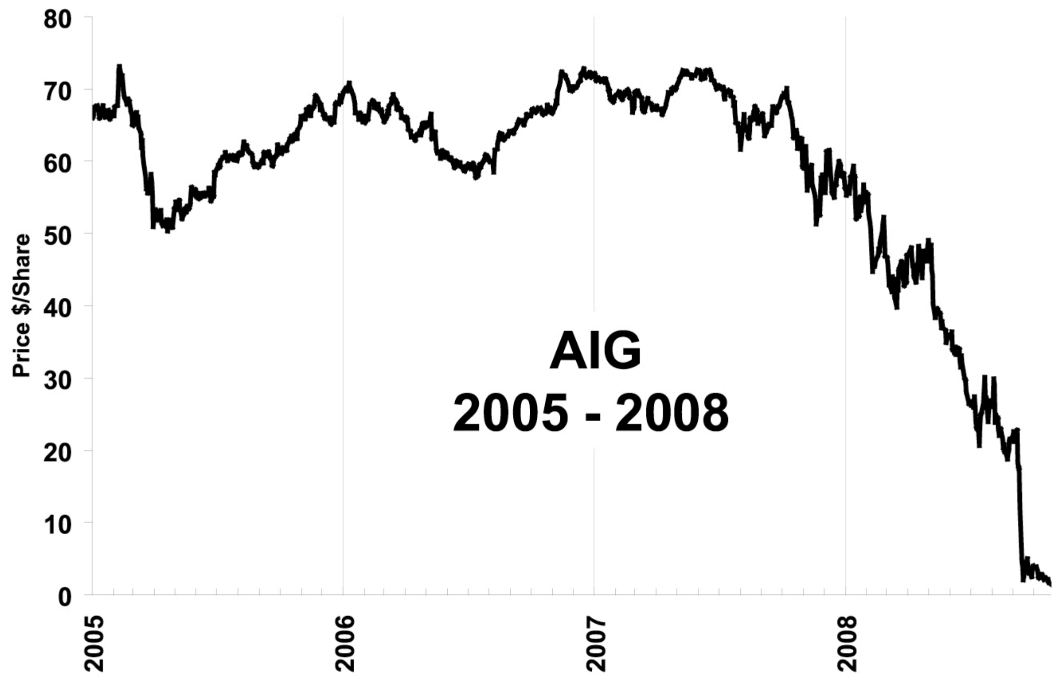

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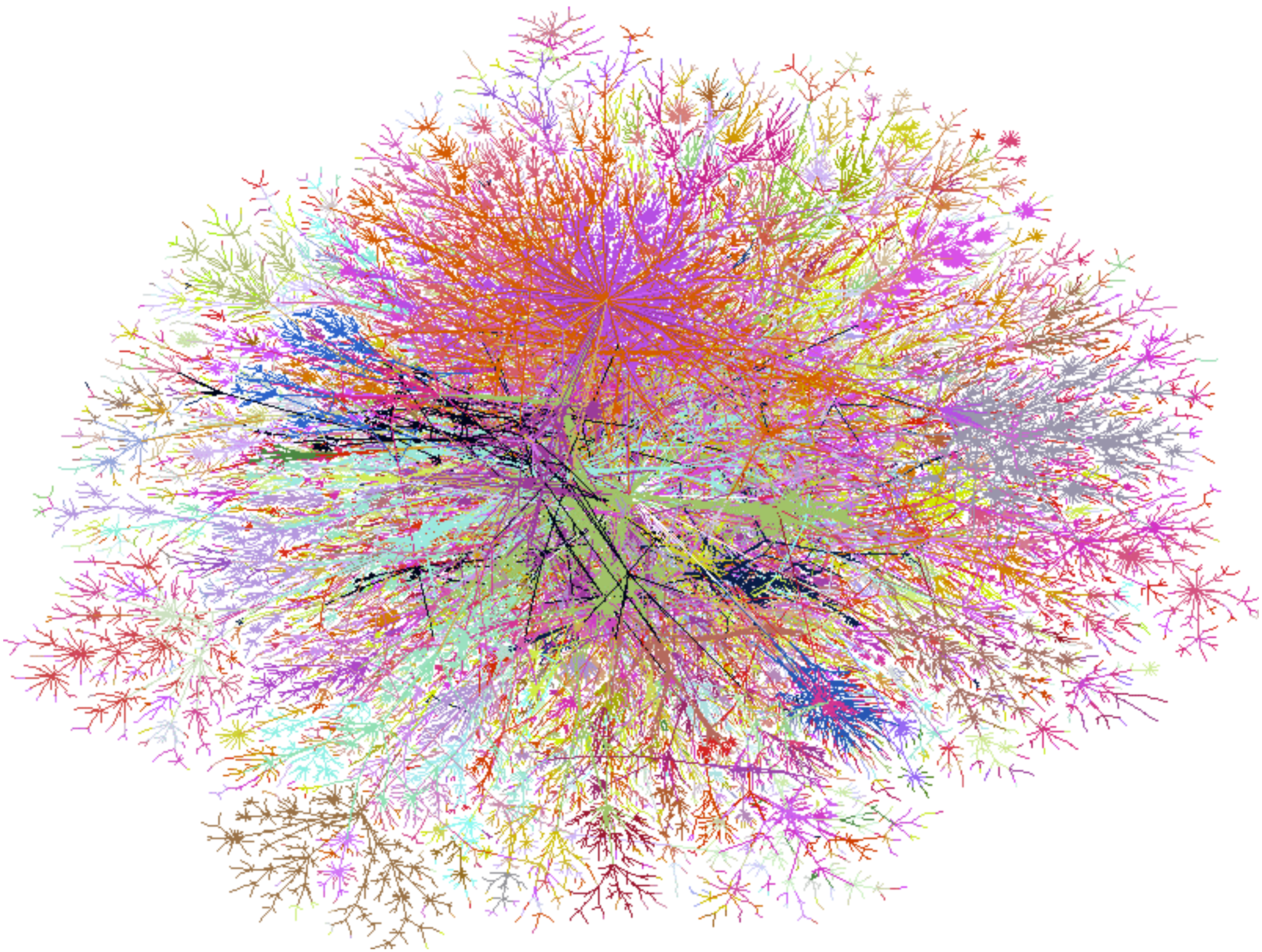


Uncertainty in Computer Science

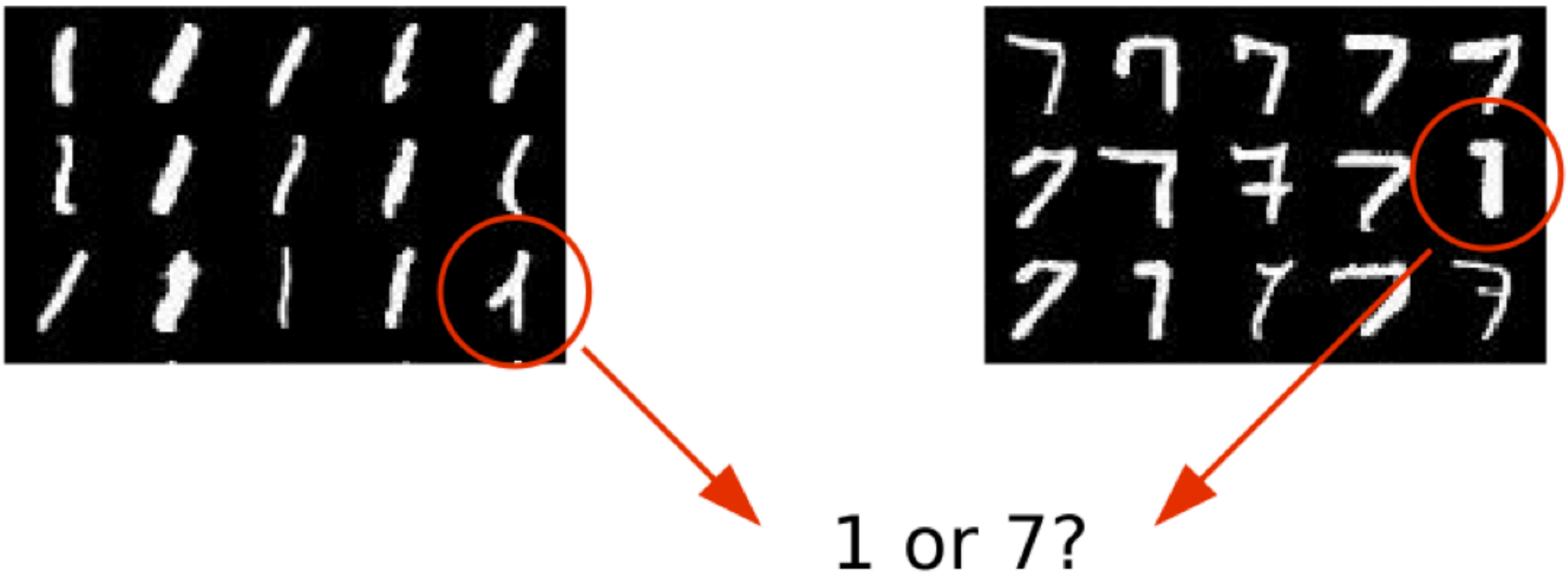
To: Jenn Wortman Vaughan
From: Jeff Vaughan
Subject: Plans for tonight ✔

To: Jenn Wortman Vaughan
From: Jens Palsberg
Subject: Meeting ✔

To: Jenn Wortman Vaughan
From: Bob Smith ✖
Subject: V14GR4 4 U

To: Jenn Wortman Vaughan
From: NIPS Committee ?
Subject: Paper decision

Uncertainty in Computer Science



Uncertainty in Computer Science

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Many areas of computer science involve uncertainty.

In this course, you will develop foundational, mathematical reasoning skills that will help you deal with it.

Course Overview
& Logistics

Part I: Formalizing Uncertainty

- Sample spaces and events
- Probability laws
- Conditional probability, Bayes' theorem, and independence
- Counting problems (combinations, permutations)
- Random variables
- Expectation, mean, and variance
- Information and entropy
- Covariance and correlation
- Limit theorems

Part II: Reasoning Under Uncertainty

Inference: How can we estimate quantities or properties of interest from observed data?

- Example: Inferring if a new email is spam or not

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Markov Chains: How can we reason about random processes that evolve over time?

- Example: Random surfer used in Google's PageRank

Course Staff

Instructor: Prof. Jenn Wortman Vaughan (jenn@cs.ucla.edu)

Office Hours: Thursdays, 11–12pm, Boelter 4532H

and by appointment

TA: Jacob Mathew (jacobgmathew@gmail.com)

Office Hours: Tuesdays, 11am–1pm, Boelter 2432

Discussions: Fridays, 2-3:50pm and 4–5:50pm

Graders: Ding Zhao (zhaoding@ucla.edu)

and Jake Stothard (stothardj@gmail.com)

Resources

Required textbook:

Introduction to Probability (2nd Edition)

by Dimitri P. Bertsekas and John N. Tsitsiklis

Other material, including reading assignments, problem sets, and lecture slides, will be posted at:

`http://www.cs.ucla.edu/~jenn/courses/S12.html`

Breakdown of Grades

Five Homework Assignments (25%)

- Mostly pencil-and-paper, a little C++ programming

In-class Exercises (15%)

- Given frequently during both lecture and discussions

Midterm (30%)

- Wednesday, May 2, sheet of hand-written notes allowed

Final Exam (30%)

- Tuesday, June 12, cumulative, same rules as midterm

Homework Policies

Grading: On each assignment, a subset of problems will be graded. Your grade will be based on these problems only.

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Academic Honesty: Collaboration is encouraged, but you must follow the academic honesty policy!

Academic Honesty

- Each student must write down his or her own solutions independently in his or her own words.

- Each student must submit a list of anyone with whom the assignment was discussed.

- All sources (internet included) must be properly credited.

- Solution sets from this course or any other course cannot be used under any circumstances.

Piazza Discussion Board

Use Piazza to…

- Discuss or ask for clarifications about material covered in class or in your reading
- Share useful references you've found
- Find other students to form study groups
- Answer questions to solidify your understanding

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The course academic honesty policy must be followed on Piazza at all times!

In general, to succeed in this class…

- Complete the required reading before class
- Attend lecture and discussion sections regularly
- Start your assignments early
- Check the course website often:

  `http://www.cs.ucla.edu/~jenn/courses/S12.html`

- Ask the course staff if you're unsure about any policies

A Two-Minute Review of Sets

Review of Sets

- A set S is a collection of objects

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- S is a subset of itself, but not a strict subset

$$\{1, 3, 5\} \subseteq S, \qquad \{1, 3, 5\} \not\subset S$$

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- The universal set Ω contains all possible elements of interest in some context

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- The empty set $\varnothing$ is the set containing no elements

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- S and T partition Ω if they are disjoint and $S \cup T = \Omega$

Algebra of Sets

If you don't feel completely comfortable with these properties, spend some time reviewing Chapter 1.1 soon…

$$S \cup T = T \cup S$$

$$(S^c)^c = S$$

$$S \cap S^c = \varnothing$$

$$S \cup \Omega = \Omega$$

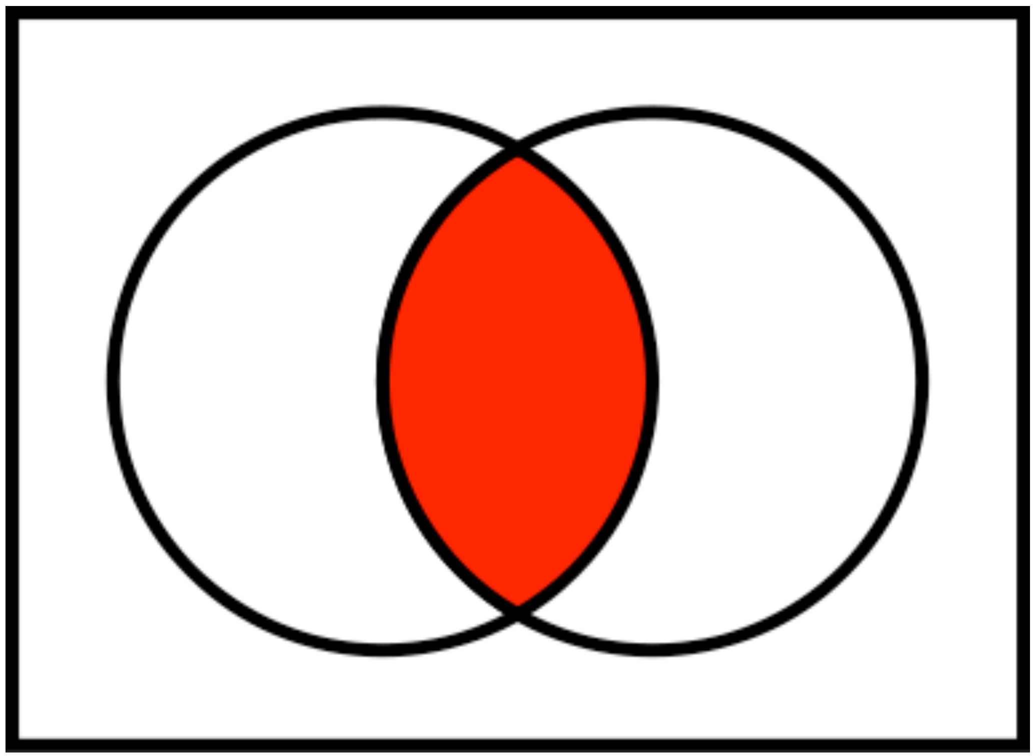
$$S \cap \Omega = S$$

$$S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$$

$$S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$$

$$S \cup (T \cup U) = (S \cup T) \cup U$$

Venn Diagrams



What is probability?

Interpretation: Relative Frequency

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What is the probability that a spaceship will remain operational during a mission to Mars? Hmmm….

Interpretation: Subjective Belief

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Can be useful when we need a principled, systematic way to make choices in the presence of uncertainty

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    - Running a computer program.

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- Elements must be exhaustive
  (we always obtain an outcome from the sample space)

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- Each of these can be represented as a set of atomic events – the basic elements of the sample space.

Events

Since events are sets, we can apply set operations to them…

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$A \cap B$ "the number is odd AND the number is ≤ 3"

Probability Law

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Together, a sample space and a probability law define a probabilistic model

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First Exercise!! Use the probability axioms and set theory to derive answers to the following questions.

- Suppose A, B, and C are disjoint. What is $P(A \cup B \cup C)$?
- What is $P(\emptyset)$?
- What is $P(A^c)$?

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- $P(A \cup B) \leq P(A) + P(B)$

These can all be verified informally using Venn diagrams.

For next time...

- Sign up for Piazza.
- Get the textbook.
- Check the website for assigned reading.

Homework 1 will be posted later this week!