CS 112: Computer System Modeling Fundamentals

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April 5, 2011

Lecture 3

Reminders & Announcements

- Pick up a copy of the course textbook
- Homework 1 is due one week from today
- There was a small typo in problem 6, so make sure you get the latest version from the website

Today

- The counting principle
- Permutations
- Combinations
- The binomial distribution
- Partitions (if we have time..)

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How can we count the items in these sets?

The Counting Principle

Consider a process consisting of r stages. Suppose that

- There are n_1 possible results at the first stage.
- Regardless of what happens at the first stage, there are n_2 possible results at the second stage.
- More generally, regardless of what happens at stage i-1, there are n_i possible results at stage i.

Then the total number of possible results for the whole process is

$$n_1 \, n_2 \, n_3 \, \ldots \, n_r$$

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- How many ways can we assign n threads to k processors in such a way that each processor is assigned exactly one thread and no thread is assigned to multiple processors? (Assume that $n \geq k$.)


AMERICA'S PLACE TO EAT.
WAFFLE HOUSE
GOOD FOOD FAST.
Our World Famous
HASHBROWNS
There are more than 1,572,864 ways to enjoy our Hashbrowns. How many have you tried? How about onions, cheese and Picante sauce?
REGULAR 1.15
LARGE 1.90
TRIPLE 2.20
SCATTERED on the grill AND...
SMOTHERED –ONIONS add .25
COVERED –CHEESE add .25
CHUNKED –"Hickory Smoked" HAM add .65
TOPPED –Bert's CHILI add .65
DICED –TOMATOES add .25
PEPPERED –Jalapeño PEPPERS add .25
NEW
CAPPED –MUSHROOMS add .65
"SCATTERED ALL THE WAY" 4.10
Minute Maid
Texas Cheesesteak Combo Plate
Includes Hashbrowns and Beverage
6.25
TEXAS Che
DOUBLE P
Chop Steak
Grilled Chi
Sausage

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Wheat Bread
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Sausage

How many optional condiments would the Waffle House need to get up to 1,572,864 ways to enjoy?

The Counting Principle

- Suppose that you have four books about math, ten books about computer science, and two books about physics. How many ways can you arrange these books on a shelf such that all of the books of a given subject are together?

Combinations

- Your favorite pizza shop offers fifteen different toppings. How many ways can you create a pizza with three distinct toppings?

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- Suppose there are ten computer science courses being offered. How many ways can you create a schedule of four courses?

Combinations

When order doesn't matter, the number of ways to choose k elements from a set of n is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

pronounced "n choose k"

Disk Failure

A system contains $2x$ disks divided into x pairs where each pair of disks contain the same data. If one of the disks in a pair fails, the data can still be recovered, but if both disks fail, it cannot.

Suppose two "random" disks fail.

What is the probability that some data is inaccessible?

Poker Hands

Consider a standard deck of 52 cards in which each card has one of four suits (hearts, diamonds, spades, or clubs) and one of 13 ranks (ace, 2, 3, …, 10, jack, queen, or king)

- Suppose you draw five cards uniformly at random from a standard deck of cards. What is the probability of drawing four cards of the same rank (kings, threes, etc.)?

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- Suppose you draw five cards uniformly at random from a standard deck of cards. What is the probability of drawing four cards of the same rank (kings, threes, etc.)?
- What is the probability of drawing five cards of the same suit (hearts, diamonds, etc.)?

Sequences of Independent Trials

- Suppose we have a biased coin that lands on heads with probability p. If we flip this coin five times, what is the probability that we see heads, heads, tails, tails, heads?

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- If we flip this coin five times, what is the probability that we see exactly three heads and two tails?

Binomial Probabilities

Consider a sequence of n independent trials, each with a "success" probability of p. The probability of any particular sequence with k successes is

$$p^k(1-p)^{n-k}$$

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Hint: Think about how these ideas apply to problem 7…

Sequences of Independent Trials

A cell phone provider has enough resources to handle up to r data requests at once. Assume that every minute, each of the provider's n customers makes a data request with probability p, independent of what the other customers do.

What is the probability that exactly x customers will make a data request during a particular minute?

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What is the probability that exactly x customers will make a data request during a particular minute?

What is the probability that more than r customers will make a data request during a particular minute?

(we stopped here in class, but I’m leaving the slides on partitions for the curious students… you can also read about this in the textbook on pages 49–51)

Partitions

There are 60 software engineers working in an office. We would like to partition the engineers into three project groups. The projects require 10, 20, and 30 engineers respectively. How many partitions of the engineers are possible?

Partitions

Suppose we are given an n-item set which we would like to partition into r disjoint subsets of size n_1, n_2, …, n_r, such that $n_1 + n_2 + \ldots + n_r = n$. The number of possible partitions is

$$\frac{n!}{n_1!n_2!n_3!\ldots n_r!}$$

When $r = 2$, this is simply "n choose n_1"

Partitions

There are 60 software engineers working in an office, including 3 senior engineers. We would like to partition the engineers into three project groups. The projects require 10, 20, and 30 engineers respectively. If engineers are "randomly" assigned to each group, what is the probability that each group contains a senior engineer?