CS 112: Computer System Modeling Fundamentals

Prof. Jenn Wortman Vaughan

May 24, 2011

Lecture 16

Reminders & Announcements

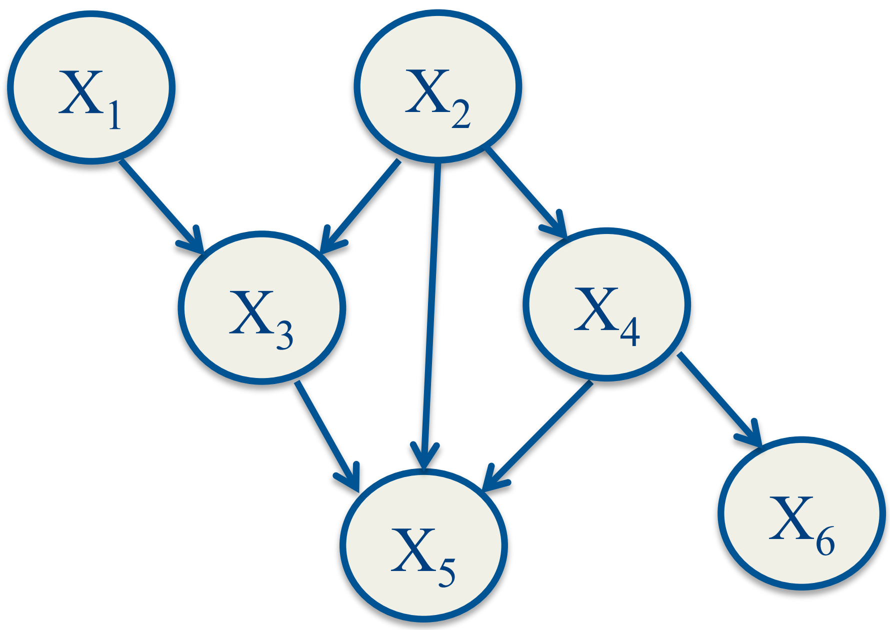
- Homework 4 is due tonight at 11:59pm
- Homework 5 will be posted by Thursday
- All reading assignments for the remainder of the quarter have been posted on the website

Today

- Bayesian Networks
  - Review of what they are
  - D-separation
  - More examples of how to work with Bayesian networks

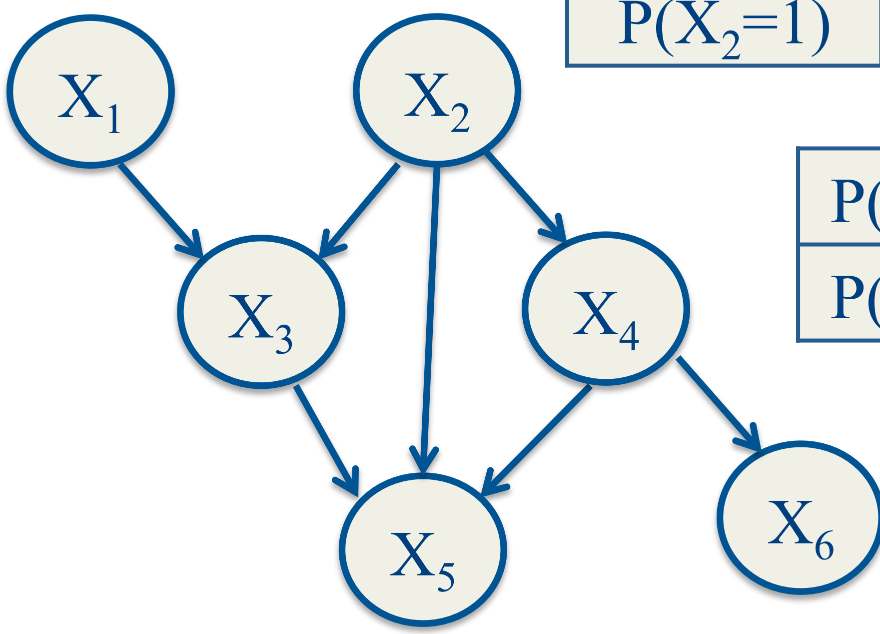
Bayesian Network

- The first component of a Bayesian network is a directed acyclic graph (or DAG)
  - Nodes represents a random variables
  - Edges represent dependencies



Bayesian Network

- The second component of a Bayesian network is a conditional probability table for each node



$P(X_2=1)$	0.7

$P(X_4=1 \mid X_2=1)$	0.6
$P(X_4=1 \mid X_2=0)$	0.3

- The joint distribution is uniquely specified by these tables

Bayesian Network

Key Assumption: If dependencies are represented accurately, the joint distribution factors in a nice way

$$P(X_1 = x_1, \ldots, X_n = x_n)$$
$$= \prod_{i=1}^{n} P\left(X_i = x_i \mid X_j = x_j \text{ for } X_j \in Pa(X_i)\right)$$

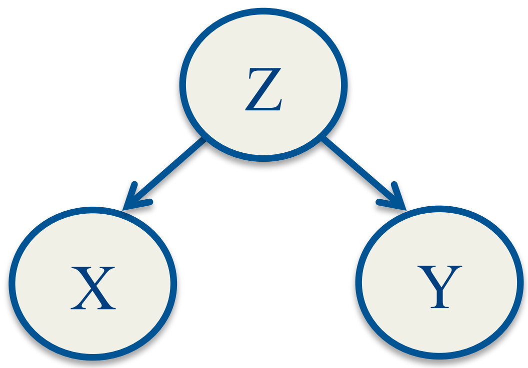
Don't need to learn or store the whole joint PMF!

Can instead learn a small conditional probability table for each random variable.

Causal Chains

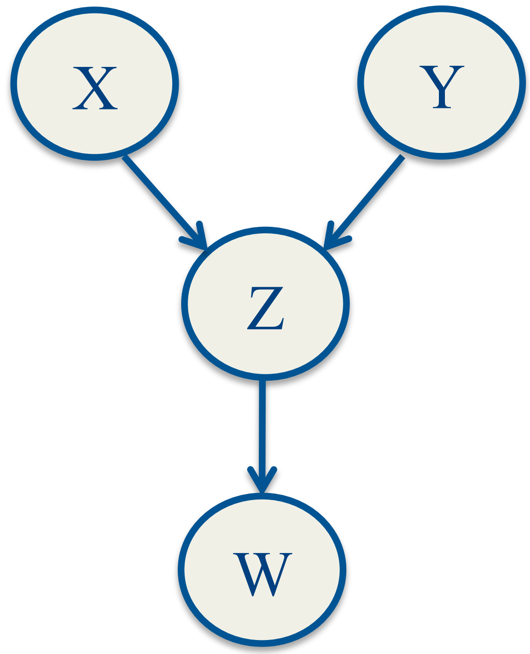
- X and Y are conditionally independent given Z
- In general, X and Y are *not* independent

Common Cause



- Again, X and Y are conditionally independent given Z
- In general, X and Y are *not* independent

Common Effect



- X and Y are independent
- X and Y are *not* conditionally independent given Z
- X and Y are *not* conditionally independent given W

Independence Relationships

- To determine if two nodes are independent, we just have to check the path (or paths) between them

- If every path is “blocked”, they are independent

More Formally...

Let *E* denote a set of observed evidence nodes. Each node in the graph is "closed" or "open" given evidence *E*.

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$\leftarrow W \rightarrow$	closed if $W \in E$
$\rightarrow W \leftarrow$	closed if neither W nor any descendent of W is in E

D-Separation

- Given evidence *E*, nodes X and Y are d-separated if **every path** between X and Y contains at least one closed node.

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- Given evidence *E*, nodes X and Y are d-separated if **every path** between X and Y contains at least one closed node.

- If X and Y are d-separated given evidence *E*, then X and Y are conditionally independent given *E*.

D-Separation

- Multiple networks can correspond to the same set of independence assumptions...

Reasoning with Bayesian Networks

- Can answer questions of interest using the law of total probability...

Quiz #4