CS 112: Computer System Modeling Fundamentals

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May 19, 2011

Lecture 15

Reminders & Announcements

Alternative grading scheme…

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- If it works out in your favor, we will count the final exam for 45% of your grade and the midterm for 15%

Joint PMFs

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Let X indicate whether or not someone has a disease

Let Y indicate whether or not his test results are positive

If we know the joint PMF, we can answer questions like:

- How likely is it that the patient has the disease?
- Given that the patient's test result is positive, how likely is it that he has the disease?

The Problem with Joint PMFs

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- Which of two diseases is most likely given that the patient has stomach pains, skin irritation, loss of appetite, negative results on test 1, positive results on tests 2 and 3, and has not yet been given test 4?

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As computer scientists, we are interested in finding *efficient* ways to reason about complex scenarios

Efficient Representations

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- Graphical models such as Bayesian networks allow us to take advantage of more complex independence relationships between variables

Bayesian Network

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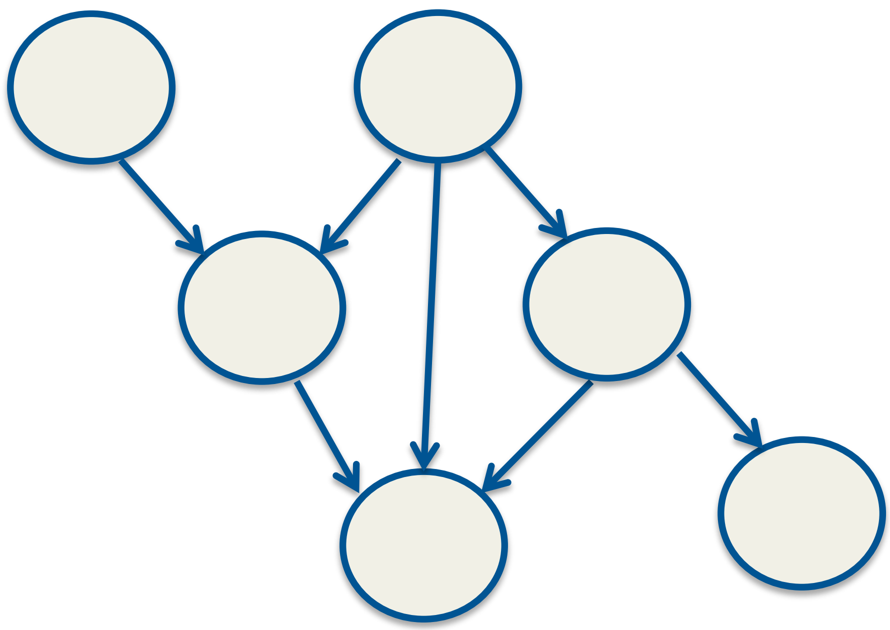
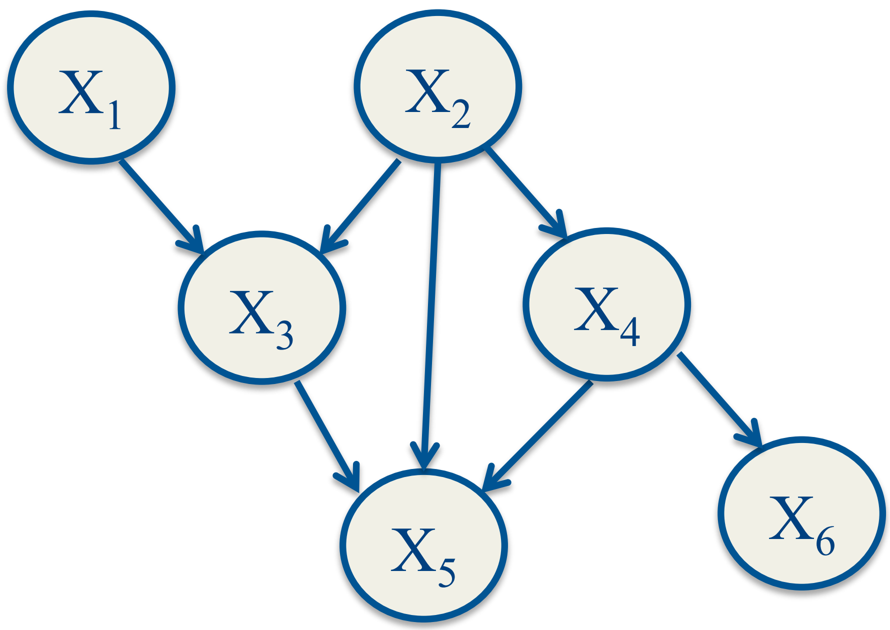


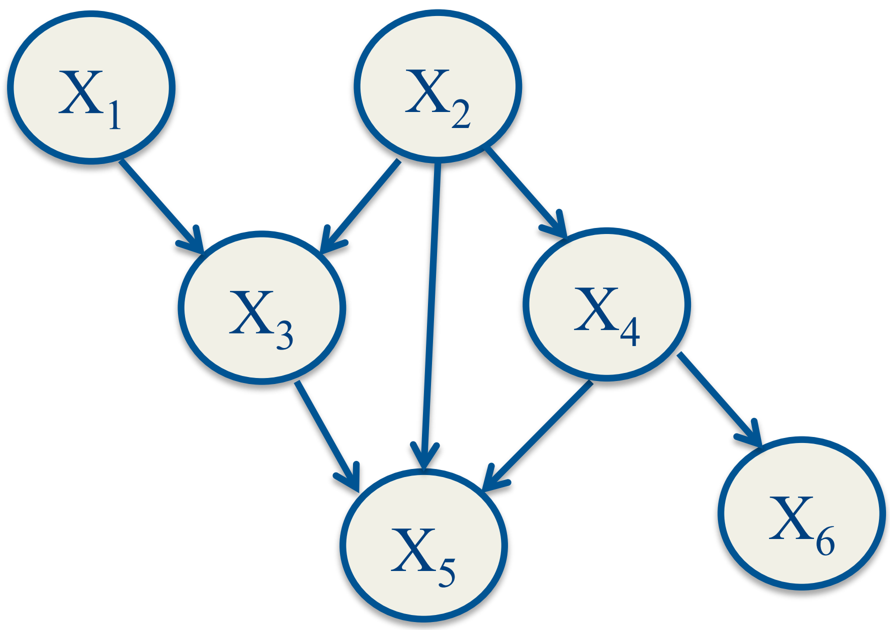
Bayesian Network

- The first component of a Bayesian network is a directed acyclic graph (or DAG)
  - Nodes represents a random variables



Bayesian Network

- The first component of a Bayesian network is a directed acyclic graph (or DAG)
  - Nodes represents a random variables
  - Edges represent dependencies



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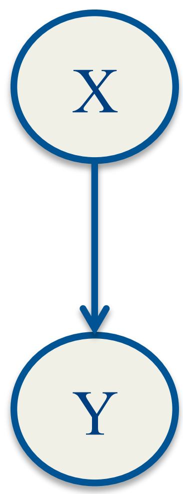

Bayesian Network

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$$P(X_1 = x_1, \ldots, X_n = x_n) = \prod_{i=1}^{n} P\left(X_i = x_i \mid Pa(X_i) = pa(X_i)\right)$$

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Don't need to learn or store the whole joint PMF!

Can instead learn a small conditional probability table for each random variable.

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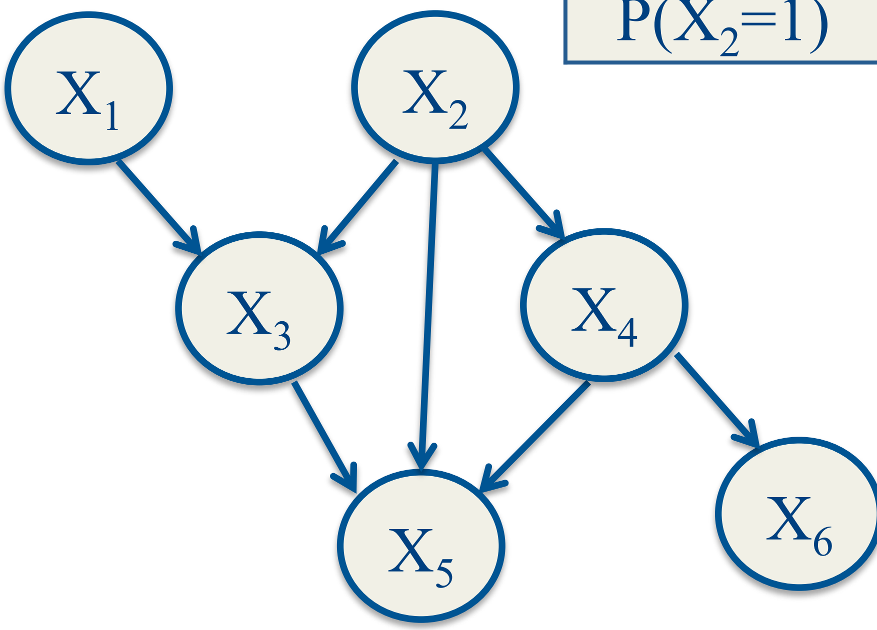
$P(X_2=1)$	0.7



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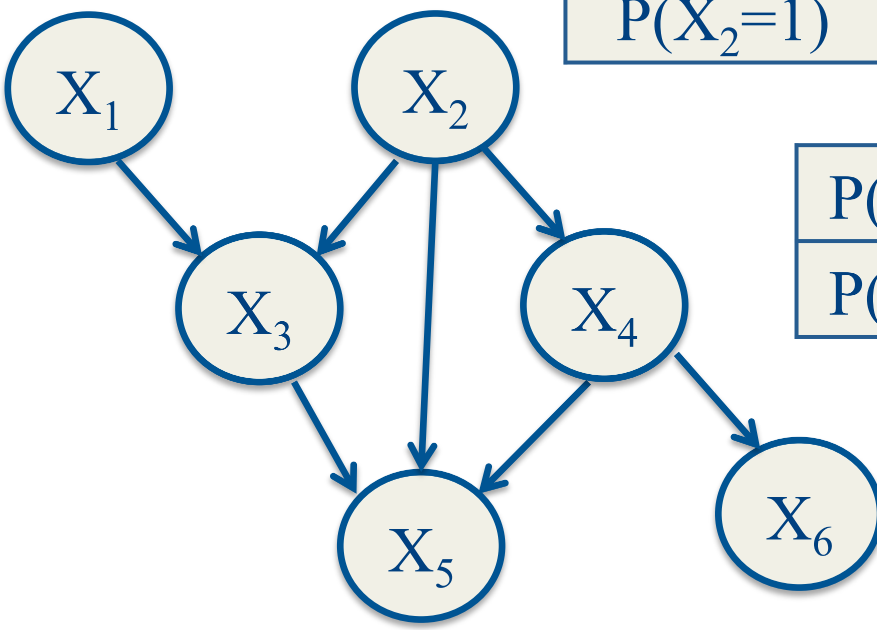

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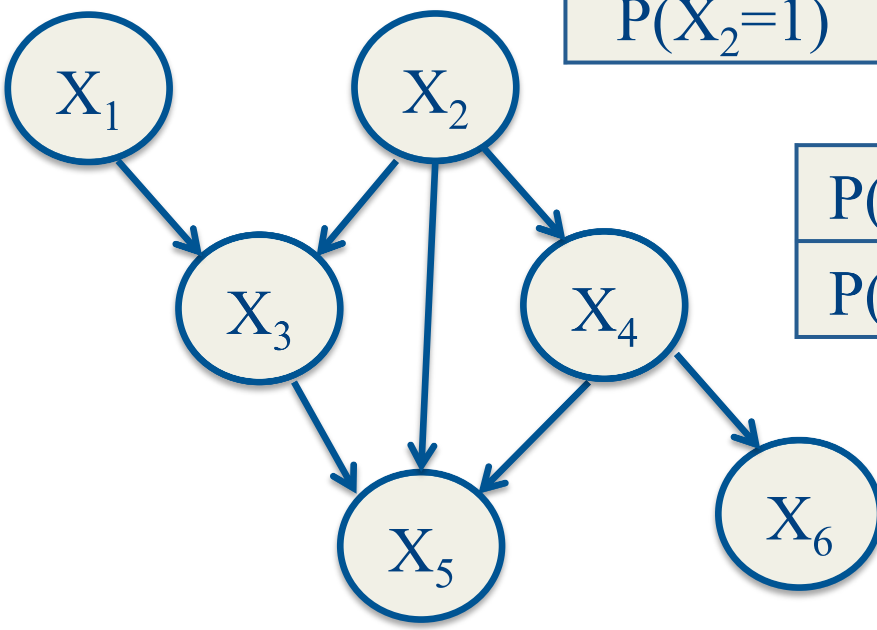

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- The joint distribution is uniquely specified by these tables

Causal Chains

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- X and Y are conditionally independent given Z

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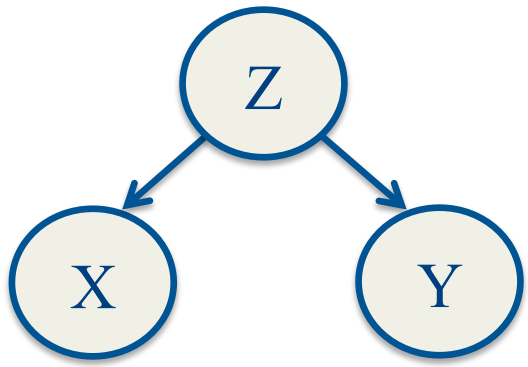
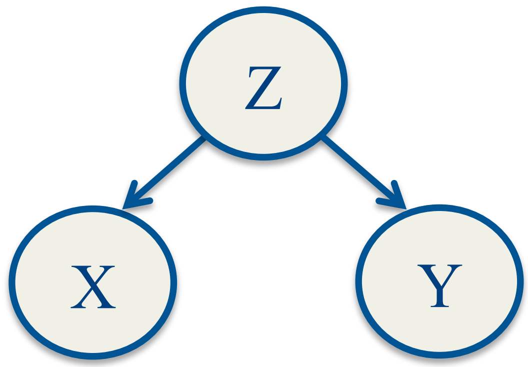
- X and Y are conditionally independent given Z
- In general, X and Y are *not* independent

Common Cause



Common Cause



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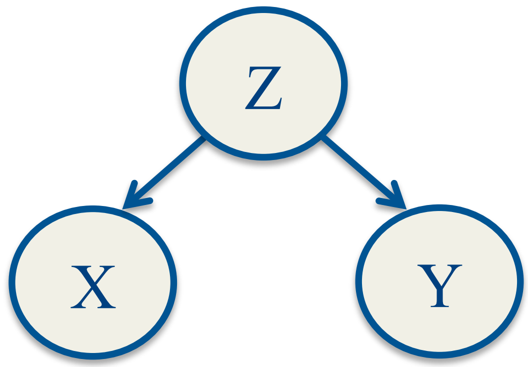

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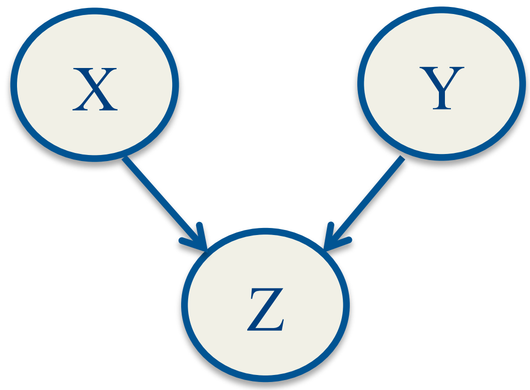
Common Effect



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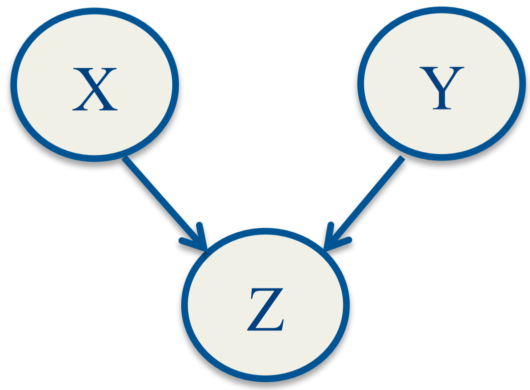

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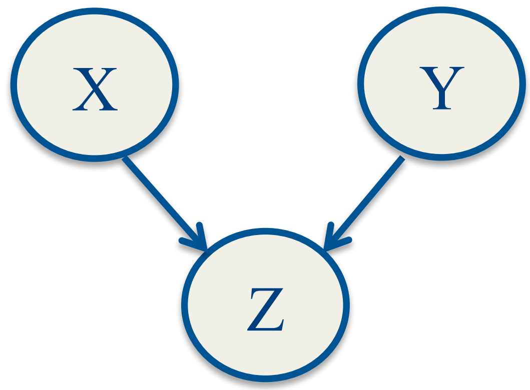
Common Effect



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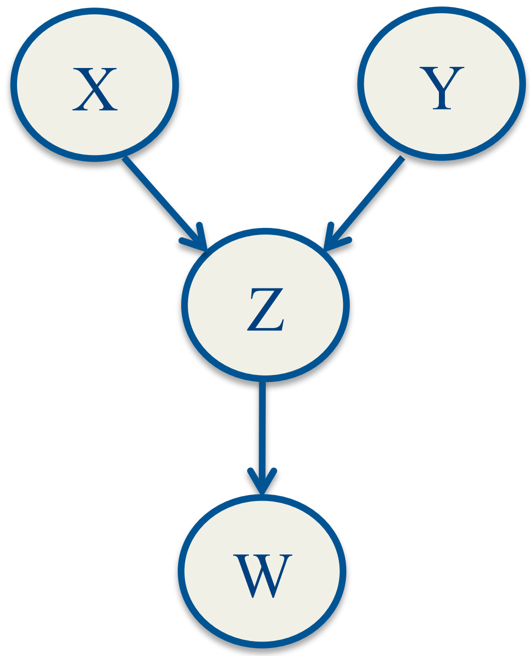


- X and Y are independent
- X and Y are *not* conditionally independent given Z
- X and Y are *not* conditionally independent given W

Independence Relationships

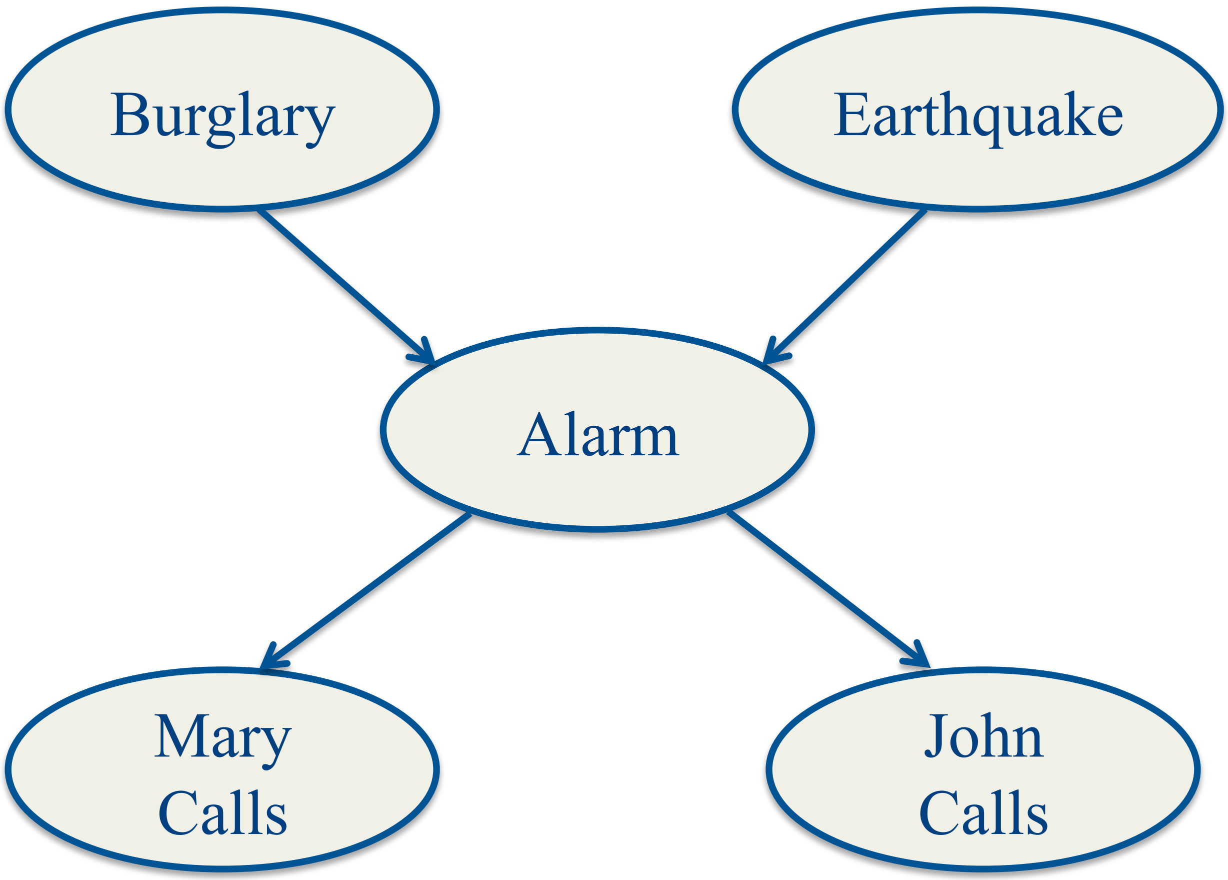
- To determine if two nodes are independent, we just have to check the path (or paths) between them
- If every path is “blocked”, they are independent

The Alarm Network

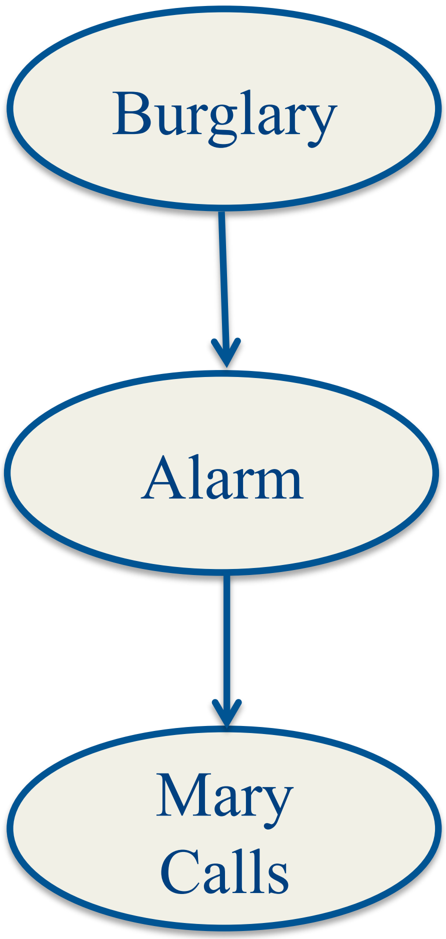
Suppose I have a new burglar alarm installed. It's pretty reliable at detecting burglaries, but sometimes responds to minor earthquakes. I have 2 neighbors (John, Mary) who promise to always call when they hear the alarm. John always calls when he hears the alarm but sometimes confuses his telephone with the alarm and calls then too. Mary likes loud music and sometimes misses the alarm.

Given some knowledge about some of these events, we'd like to reason about some others.

The Alarm Network

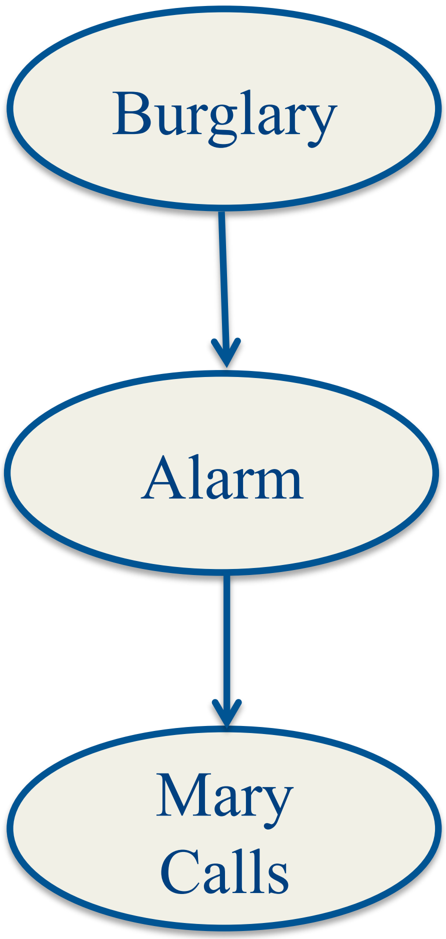


The Alarm Network



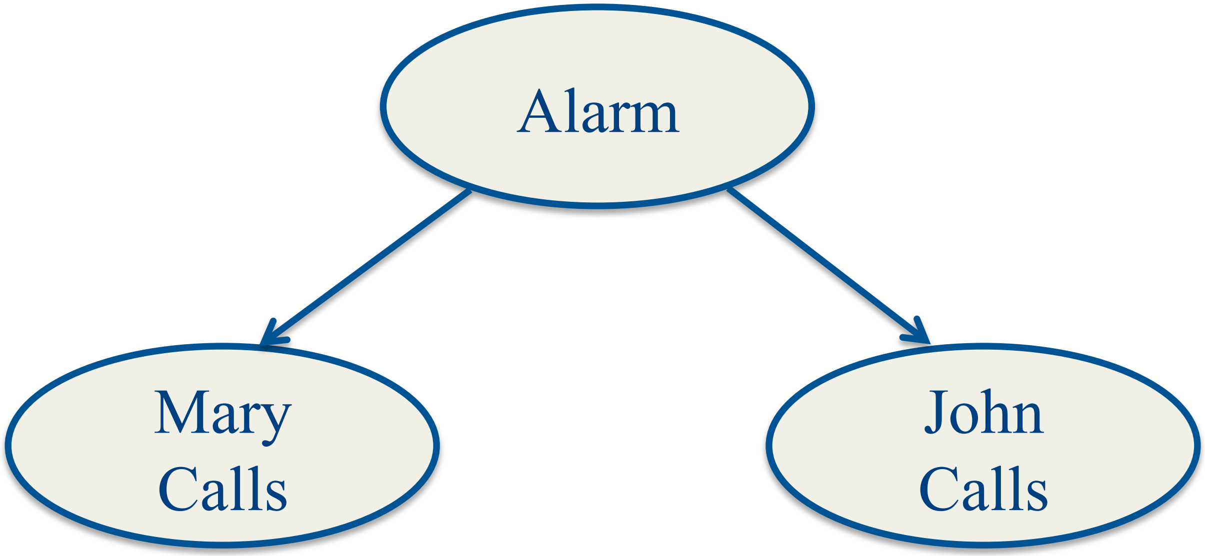
- Is a burglary independent of Mary calling?

The Alarm Network



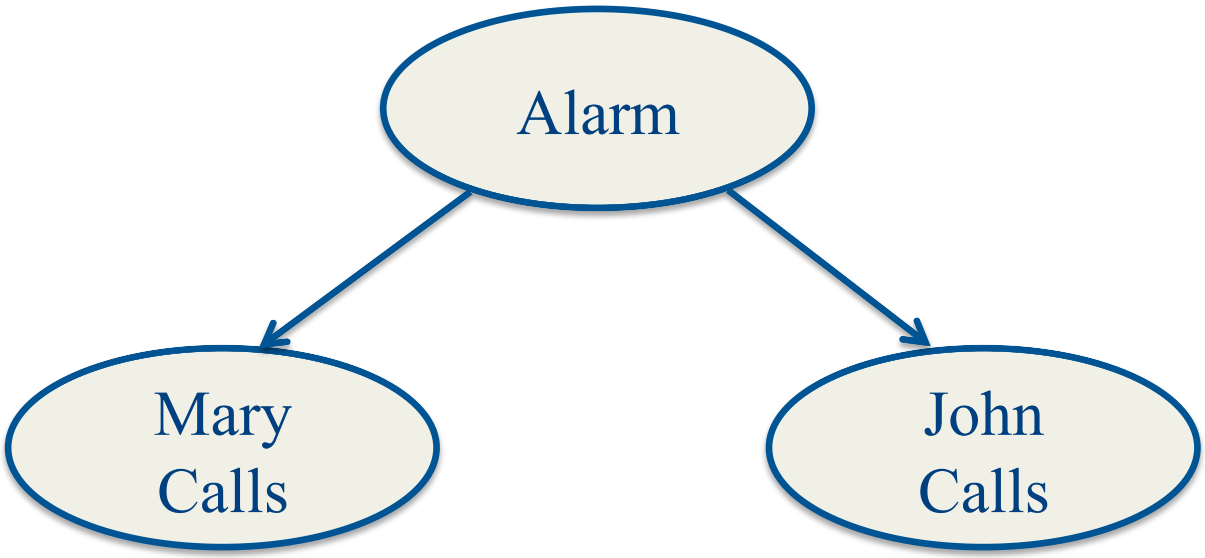
- Is a burglary independent of Mary calling?
- What if we know that the alarm went off?

The Alarm Network



- Is Mary calling independent of John calling?

The Alarm Network



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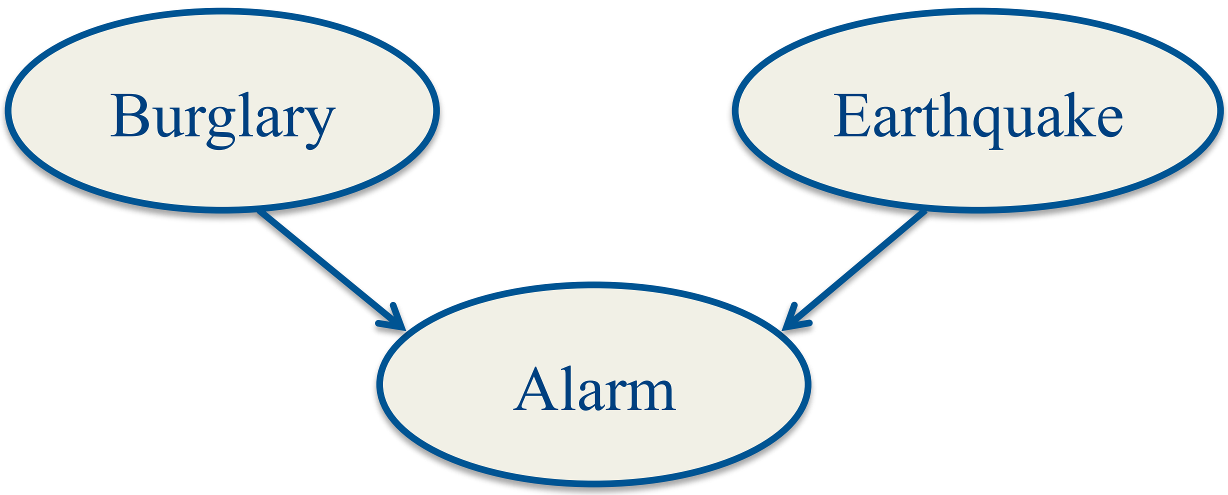
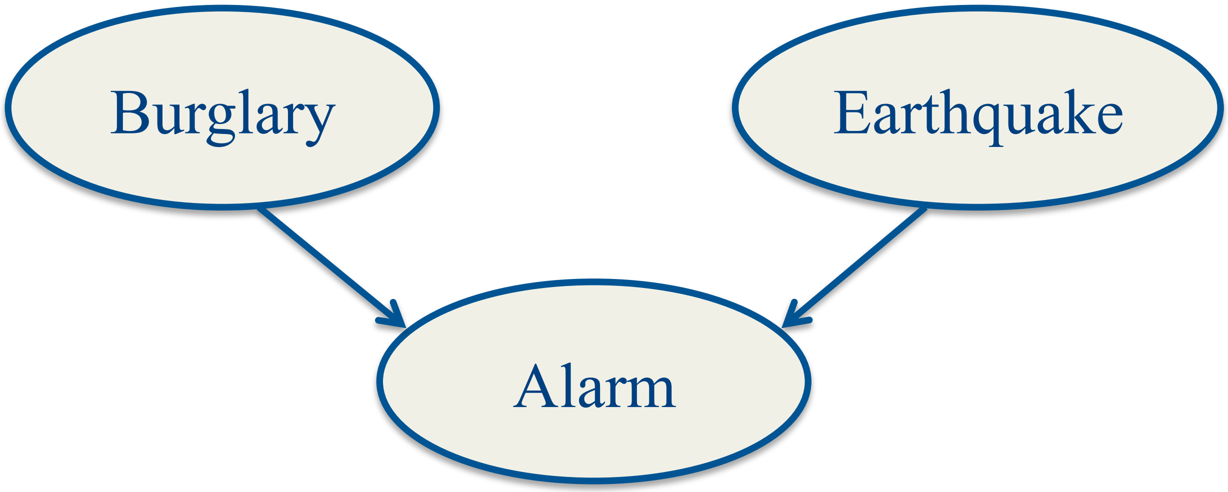


- Is a burglary independent of an earthquake?

The Alarm Network



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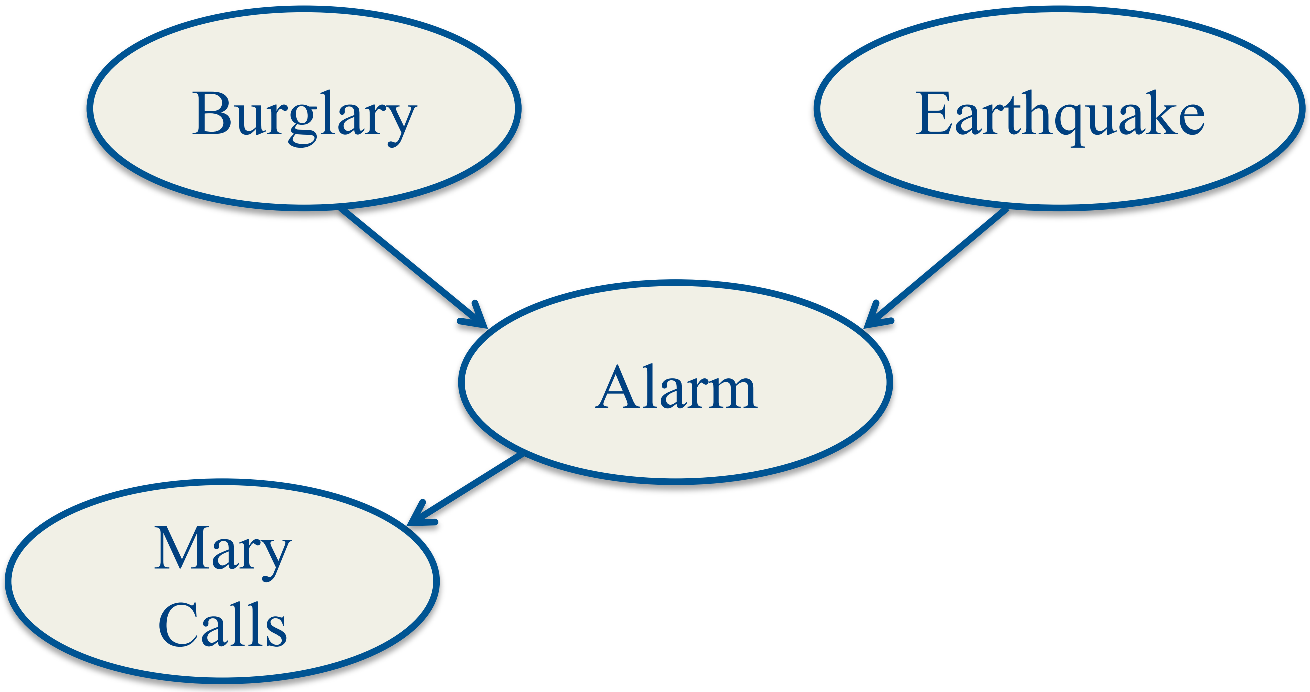
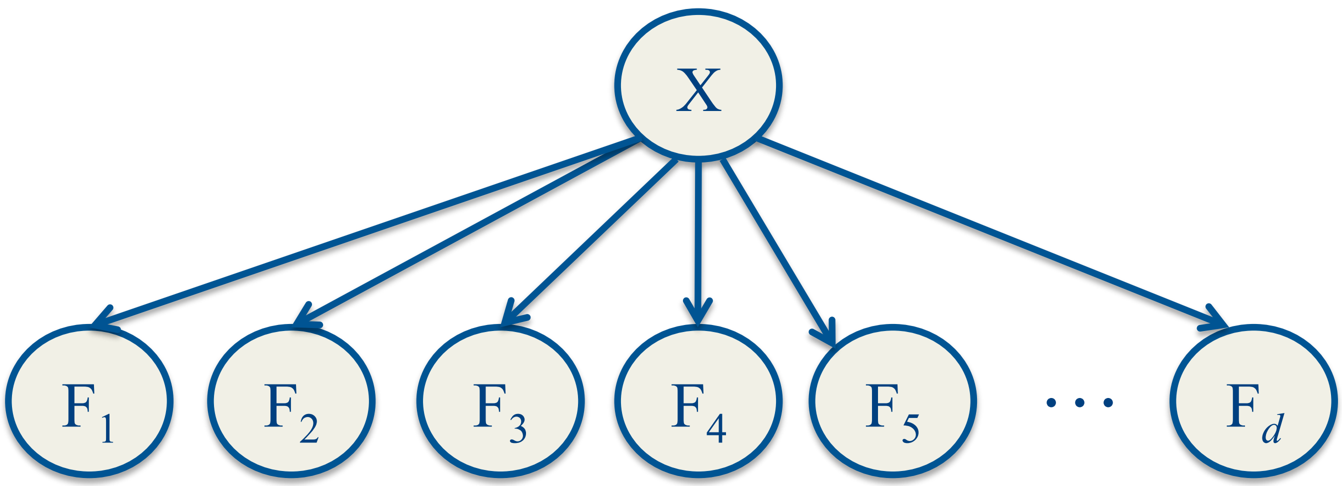
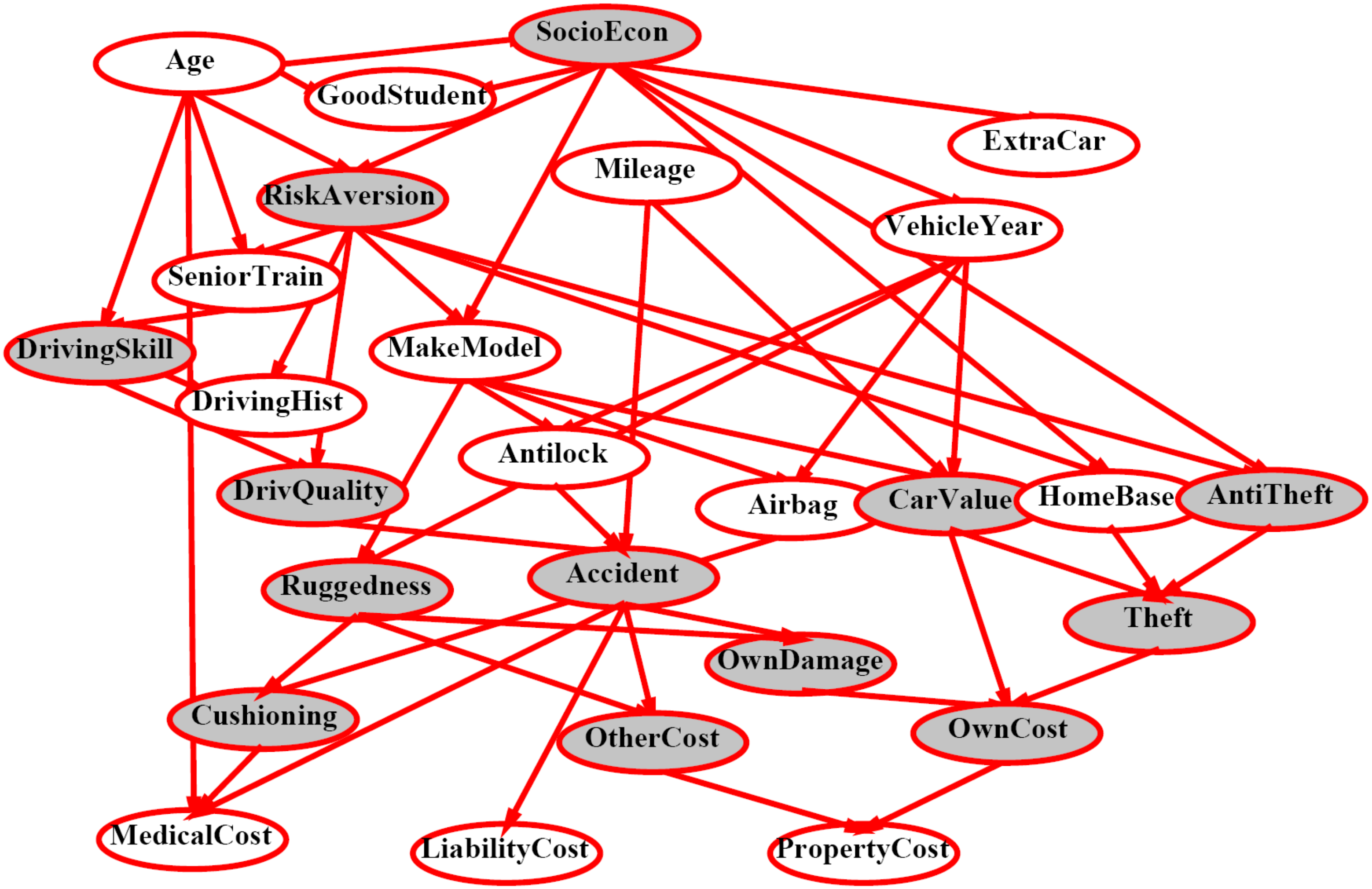


- Is a burglary independent of an earthquake?
- What if we know that the alarm went off?
- What if we know that Mary called?

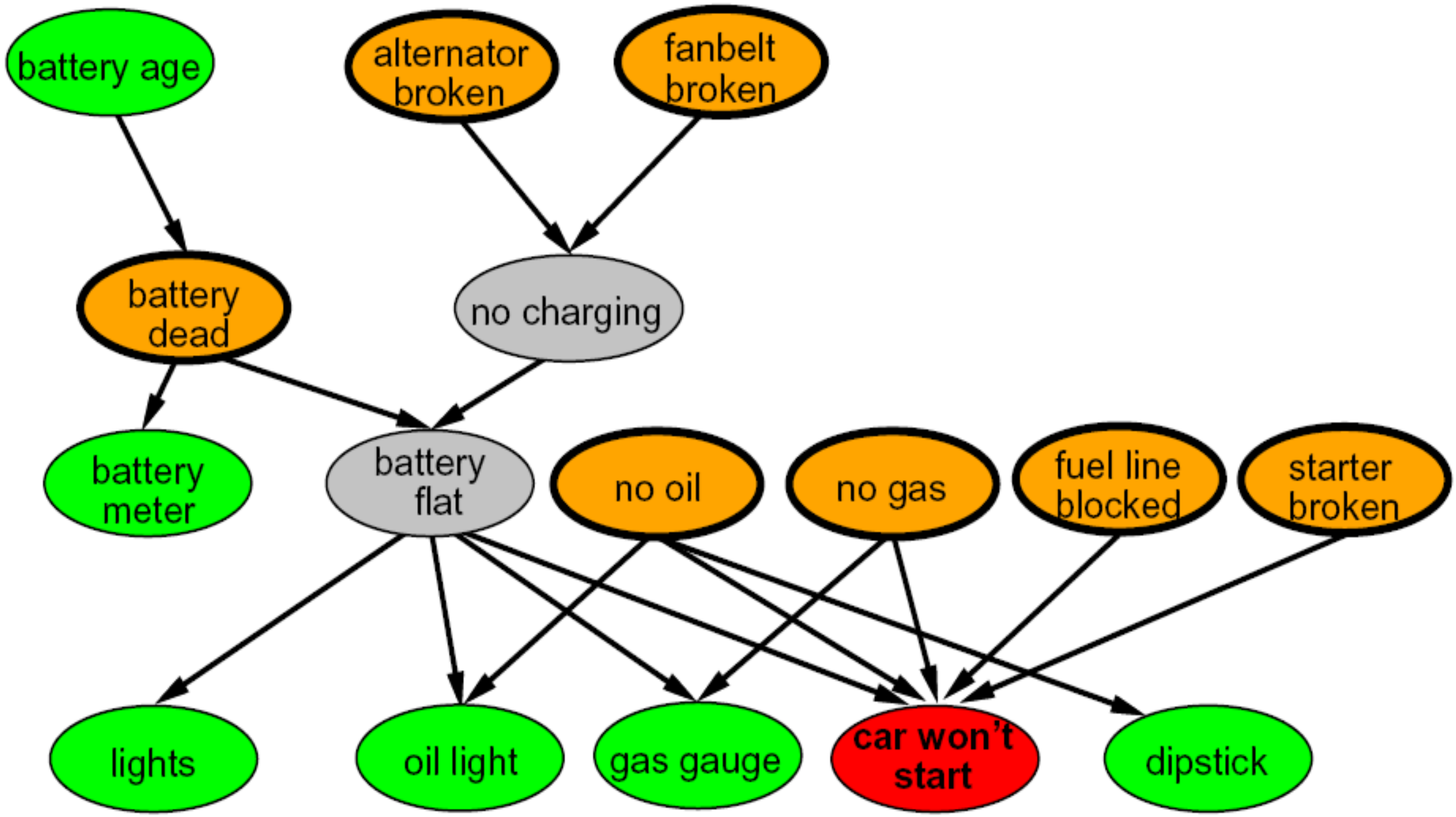
Naive Bayes Network



Insurance Network



Car Network


battery age
alternator broken
fanbelt broken
battery dead
no charging
battery meter
battery flat
no oil
no gas
fuel line blocked
starter broken
lights
oil light
gas gauge
car won't start
dipstick

Other Common Applications

- Medical diagnostics
- Bioinformatics
- Computer vision (e.g., object recognition)
- Text analysis (e.g., discovery of common topics in text)
- Time series analysis (e.g., analyzing stock market data)

- Any application in which there are lots of random variables and lots of structure