“Reasoning About Uncertainty”

CS 112: Computer System Modeling Fundamentals

Prof. Jenn Wortman Vaughan
March 29, 2011
Uncertainty in Computer Science
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To: Jenn Wortman Vaughan
From: Jeff Vaughan
Subject: Plans for tonight

To: Jenn Wortman Vaughan
From: Jens Palsberg
Subject: Meeting

To: Jenn Wortman Vaughan
From: Bob Smith
Subject: V14GR4 4 U

To: Jenn Wortman Vaughan
From: NIPS Committee
Subject: Paper decision
Uncertainty in Computer Science
Uncertainty is everywhere in computer science.

In this course, you will develop foundational, mathematical reasoning skills that will help you deal with it.
Course Overview
Part I: Probability Theory

• Sample space and events
• Probability laws and their basic properties
• Conditional probability and independence
• Bayes’ rule
• Counting problems
• Random variables
• Expectation, mean, and variance
• Covariance and correlation
• Limit theorems
Part II: On to something new!

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  • Example: Estimating parameters of a spam filter
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**Markov Chains:** How can we reason about random processes that evolve over time?
- Example: Random surfer used in Google’s PageRank

**Statistical Inference:** How can we estimate properties of random variables or model parameters from observations?
- Example: Estimating parameters of a spam filter
- Might cover some basic ideas from machine learning if we have time...
Course Logistics
Course Staff

Instructor: Jenn Wortman Vaughan (jenn@cs.ucla.edu)
Office Hours: Wednesday, 1–3pm, Boelter 4532H
No office hours this week!

TA: Ethan Schreiber (ethan@cs.ucla.edu)
Office Hours: Monday, 11:30–1:30, Boelter 2432
Discussions: Friday, 4–5:50pm, in this room

Grader: Brian Geffon (briangeffon@gmail.com)
Textbook and Material

Required textbook:

Introduction to Probability (2nd Edition)
by Dimitri P. Bertsekas and John N. Tsitsiklis

Other materials, including reading assignments, problem sets, and any lecture slides, will be posted at:

Breakdown of Grades

Five Homework Assignments (30%)
  • Mostly pencil-and-paper, a little programming

In-class Quizzes (10%)
  • Given at the start of class, lowest quiz dropped

Midterm (30%)
  • Tuesday, May 3, one sheet of hand-written notes allowed

Final Exam (30%)
  • Thursday, June 9, cumulative, same rules as midterm
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**Academic Honesty:** Collaboration is encouraged, but you must follow the academic honesty policy!
Academic Honesty

• Each student must write down his or her own solutions independently in his or her own words.

• Each student must submit a list of anyone with whom the assignment was discussed.

• All sources (internet included) must be properly credited.

• Solution sets from this course or any other course cannot be used under any circumstances.
What is probability?
Interpretation: Relative Frequency

Example: If we say a well-manufactured coin lands on heads with probability 0.5, we might mean that if we flip the coin a large (infinite) number of times, the coin should land on heads about half the time.
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What is the probability that a card randomly selected from a deck is a spade?

What is the probability that a spaceship will remain operational during a mission to Mars? Hmmm….
Interpretation: Subjective Belief

Example: A detective believes that a particular suspect committed the crime with probability 0.7
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Can be useful when we need a principled, systematic way to make choices in the presence of uncertainty
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Related to gambling odds…
Basic Definitions

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  - Flipping a single coin.
  - Flipping two coins in a row. (Multiple alternatives..)
  - Pulling a random card from a deck.
  - Running a computer program.
Samples Spaces

There can be more than one right way to define the sample space for a particular experiment.

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Always has to satisfy two properties:

- Elements must be mutually exclusive
  (only one can be the real outcome)
- Elements must be exhaustive
  (we always obtain an outcome from the sample space)
Events

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• Each of these can be represented as a set of atomic events – the basic elements of the sample space.
Events

Since events are sets, we can apply set operations to them...

\[ A = \{1,3,5\} \quad \text{“the number is odd”} \]
\[ B = \{1,2,3\} \quad \text{“the number is } \leq 3\text{”} \]
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\[ A \cap B \quad \text{“the number is odd AND the number is \leq 3”} \]
Probability Axioms

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• What is \( P(A^c) \)?
Other Properties of Probability Laws

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Challenge: Try to prove these properties on your own using the axioms of probability and the rules of set theory!
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• Can show that conditional probabilities satisfy our 3 axioms with B playing the role of \( \Omega \)